

β^+ and electron capture decay of fp shell nuclei with $Tz = -2$

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Allowed β^+ branches of very proton-rich fp shell $Tz = -2$ nuclei at the proton drip-line are calculated in the full fp valence space. The β^+ decay half-lives calculated with the standard quenching factor (g_A^{eff}/g_A)=0.74 are in good agreement with existing experimental data. Detailed branching Gamow-Teller strength are predicted but comparison with experiment is still difficult since, in most cases, spectroscopic information is not yet available.

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I. INTRODUCTION

The continuous development of experimental techniques and in particular those of Radioactive Ion Beam facilities allow now studies at the drip lines. Since Coulomb interaction strongly limits spatial extension of proton rich nuclei, the limit of stability on this side of the nuclear chart is closer to accessibility. Moreover, going on the proton rich side, Q_{β^+} energy window increases, giving access to large Gamow-Teller strength. This latter is a very sensitive test of the microscopic structure of many-body wave functions, but also of the different correlations which can reduce the observable strength.

In the sd shell, Muto et al. [1] have studied the β^+ decay of several proton rich nuclei. With the Wildenthal interaction [2], they nicely reproduced half lives for nuclei in the range of $T_z = -1$ to $T_z = -3$. A global quenching factor of 0.60 was used for GT transitions. Nevertheless, deviation of this overall value was observed by direct comparison of strength distribution up to the giant resonance region. This behaviour, also observed in the fp shell was recently analysed [3] as the influence at high energy of intruder strength mixing.

Ormand [4] has also recently investigated properties of proton drip-line nuclei, at $sd - fp$ interface. Calculated β decay half lives were in overall good agreement with available experimental ones. Predicted branching ratios for β decay were given. Particular attention was made to determine the best candidates for the observation of correlated two-proton emission. These turned to be ^{38}Ti and ^{45}Fe . Quite recently, these calculations have been extended by Cole [5] to both heavier and lighter nuclei. As a result, ^{34}Ca , ^{48}Ni and ^{54}Zn have also be proposed as nuclei unstable to diproton decay.

Theoretical studies give a microscopic description and

a successful account of many observables for stable or unstable fp shell nuclei. In particular, detailed shell model calculation of the GT strength for $A = 48, 47 - 49, 54$ and 56 [16–18] have been made and a good agreement has been found with the available (p,n) data. In the line of these studies, we investigate now the GT strength through the weak probe, looking into the beta decay properties of $Tz = -2$ nuclei from ^{44}Cr to ^{56}Zn (see Figure 1). As ^{54}Cu is predicted to be unstable, it was not included in our study.

Recent significant advances have been made in the spectroscopy of these very proton-rich nuclei [20–23]. But if half-lives could be measured in all cases, except ^{56}Zn , the branching strength are still poorly known. With the available data, the analysis of the proton spectra reveals the decay of the IAS, following the Fermi beta transition, with in addition one or two peaks which describe only partly the decay of the states populated through Gamow-Teller transitions. Information on gamma transitions subsequent to, or in competition with the proton emission of the beta populated states, is not yet available.

II. VALENCE SPACE, OPERATORS AND PROCEDURE

We consider the fp shell for protons and neutrons. All the calculations are done in the full space up to the case of the decay ^{48}Fe . For $A = 50, 52$ and 56 , we calculate the ground state and the Gamow Teller sum rule in the full space, but we need to introduce a truncation scheme for the study of the detailed strength function. With the usual notations where f stands for $f_{7/2}$, r for any of the other subshells ($p_{1/2}, p_{3/2}, f_{5/2}$) and t for the number of

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particle excitations in the upper shells, we restrict ourselves to configurations “at the level t ” which includes configurations :

$$f^{n-n_0} r^{n_0} + f^{n-n_0-1} r^{n_0+1} + \dots + f^{n-n_0-t} r^{n_0+t}.$$

At the level of truncation “ t ”, we keep in the space only configurations with a maximum of t particles promoted from $f_{7/2}$ to $p_{1/2}, p_{3/2}, f_{5/2}$ shells. We can remark that the σ operator which connects spin-orbit partners ($f_{7/2}$ and $f_{5/2}$) introduces configurations at the level $t+1$. Moreover, T_+ operator for $Z > 28$ also introduces configurations at the level $t+1$.

We use the KB3 interaction [6], a minimally modified version of Kuo-Brown interaction. This latter interaction was cured to fix correct single particle energies around ^{48}Ca and ^{56}Ni . As already mentioned, it gives an excellent agreement with spectroscopic data in the region of stable nuclei. Shell model matrices are built and diagonalized with the code ANTOINE (m-scheme) [13] and a new shell model code operating in coupled scheme representation [14,15]. Gamow-Teller operators are renormalized by the standard value in the fp shell : $(0.736)^2$ corresponding to the systematics of [7] and taking into account the recent measurement of ratio of axial over vector coupling constants [25],

$$\begin{aligned} B(GT) &= \left(\frac{g_A}{g_V} \right)_{eff}^2 \times \langle \sigma \tau \rangle^2 \\ \text{with } \left(\frac{g_A}{g_V} \right)_{eff} &= (0.736) \times \left(\frac{g_A}{g_V} \right)_{free} \\ \text{and } \left(\frac{g_A}{g_V} \right)_{free} &= -1.274(3). \end{aligned}$$

The half-lives are given by :

$$(f^A + f^e)t = \frac{6146 \pm 6}{\left(\frac{f_V}{f_A} \right) B(F) + B(GT)},$$

$$\text{with here } B(F) = \langle \tau \rangle^2 = [T(T+1) - T_{zi}T_{zf}]$$

The ground state Q_{EC} values are obtained with the atomic masses taken from the compilation by Audi and Wapstra [8]. For these nuclei, far from stability, reported values (and errors) result from systematic trends. For the evaluation of the Fermi transitions, the energy difference between pairs of isobaric analog levels is obtained with the Coulomb displacement energies recently tabulated by Antony et al. [9]. In this work, for levels with isobaric spin $T = 2$, fits to the most recent data yield to the equation:

$$\Delta E_C = 1406.7(6) \frac{Z}{A^{\frac{1}{3}}} - 872.8(32)$$

where Z is the average charge of the pair and A the mass number.

III. B(GT) RESULTS

Gamow-Teller sum rule S^+ , calculated in the full space are given in table[1]. It can be compared to S^- in the mirror nucleus given by charge exchange reactions (p, n) [10,11] which make possible to observe, in principle, the total Gamow-Teller strength distribution.

The increase of S^+ with the mass number, reported in Table 1, corresponds to the expected variation in this part of the fp shell as the number of valence protons increases with A and the number of neutron holes is high. If we plot the calculated S^+ values versus the product of protons, $Z_{val} = Z - 20$ and the number of neutron holes in the full fp shell, $40 - N$ (see Figure 2), the value increases, roughly linearly, as shown already by Koonin and Langanke [12] who discussed the experimental results on S^+ in mid- fp shell nuclei.

Strength functions are obtained with the Whitehead method [24] by a Lanczos procedure on an initial state taken as the sum rule state $|\Sigma\rangle = \sigma \tau_+ |\psi_{initial}\rangle$. This produces the splitting of the sum rule state over physical states in the daughter nucleus. To approximate Gamow-Teller strength distribution with enough accuracy, we need to perform a large number of Lanczos iterations. For each decay of the calculation, we have performed 300 iterations, which allow a correct convergence of all low-lying eigenstates taking part into the decay. Due to this procedure and to large dimensionalities in $A = 50, 52$ and 56 , strength functions were performed in a restricted configurations space corresponding to $t = 4$.

In Figure 3, the $B(GT)$ distribution is shown. It reveals a very high density of transitions, especially in the case of the decay of the odd-odd nuclei ^{46}Mn and ^{50}Co , where the initial non zero angular momentum is connected to three different final values. A resonant-like structure is appearing with a maximum below the Q_{EC} limit. It should be noticed that most of these daughter states are proton unbound and that the observation should be reached by beta-delayed proton delayed spectroscopy. Gamma detection should also be associated to take into account the radiative width of the levels populated in these decays.

IV. BETA DECAY LIFETIMES

Half-lives were obtained from the partial lives of each GT decay and from Fermi decay partial life. Figure 4 details the beta branching ratio of each decay i.e. the relative contribution of each decay to the total half-life. Attention was paid to reach convergence for each contribution in the cases of even-even parent decay. An arbitrarily cut off on the vertical scale was set at 10^{-4} and could represent the experimental sensitivity.

If we consider the general features of the decays we have studied, we note first the significant contribution of the Fermi transition in the decay rate, in particular for the even-even nuclei. We also note (see fig. 4) the strong decrease of the beta-branch intensity with the excitation energy in the daughter nucleus. This fall introduces an experimental limit, far below the Q_{EC} value.

The theoretical calculation and experimental values are compared in table [2]. Except for ^{56}Zn , half-lives have been measured and are found in good agreement with shell model estimates, in particular for even-even nuclei.

For comparison, we have also reported two different predictions: the results of the gross theory of beta decay of Tachibana et al. [26] for a given set of their parametrization and also the recent evaluation of the half-life with respect to Gamow-Teller β decay calculated by P. Möller et al. [27] from a Quasi-particle Random Phase Approximation. The QRPA estimates give a good agreement for ^{48}Fe and ^{50}Co but large deviations are found in the other decays. Tachibana model is of macroscopic nature involving a set of dedicated parameters and gives in this mass region a satisfactory description of the decay lifetimes.

In most cases of proton rich nuclei, the large Q value allows the daughter nucleus to decay mainly via proton emission. Nevertheless, in the particular case of the IAS of even-even parent, we have computed $M1$ transitions and gamma widths. These results are shown in table III. The strongest transition between $0^+(T = 2)$ and $1^+(T = 1)$ is only reported here and the gamma energy is taken from the mirror nucleus. It is found that the calculated strength are close to the upper limit which has been observed for $M1$ isovector transitions [28]. In previous experimental work [20,23,19], a deficit in the intensity of the emitted protons from the IAS was reported for the case of ^{52}Ni . Gamma deexcitation are competing with particle emission and an accurate determination of isospin violation in the proton decay with INC interaction as performed previously by Ormand [4] would allow to determine the relative intensity of the two processes.

V. CONCLUSION

The strength function, branching ratios and half-life of the $T_z = -2$ nuclei ($44 \leq A \leq 56$) have been computed in a full major oscillator shell. The calculation gives a prediction of the distribution of the Gamow-Teller strength with a maximum below, or very close to the Q_{EC} window limit. The experimental data available do not allow to test the calculated distributions but, only to compare half-lives. The comparison with other calculations like models based on the finite-range droplet model made for a very large range of nuclei, is of interest to test the reliability of the calculations for nuclei far from stability, and their sensitivity to the details of nuclear structure.

It is important to note that the same shell model calculations give an account of the Gamow-Teller strength observed in the (p, n) reactions on $T_z = +2$ targets and the one resulting from the beta decay of $T_z = -2$ nuclei. The good agreement between the calculated values and experimental ones, observed in the (p, n) data of $A = 56$, cannot be tested up to now for the beta decay.

We conclude with a few remarks concerning the problems related to the experimental study of these very proton-rich nuclei. First, the calculations indicate a strong splitting of the strength which makes more difficult the observation of the corresponding branches and impose severe limits for the background. The second remark concerns the range of the detectable beta branches which is more limited than the Q_{EC} window and makes essentially unobservable transitions in the last fraction of this window. Only a dramatic improvement in the production rates of these very proton-rich nuclei would modify this limitation and make possible the observation of the shape of the main components of the Gamow-Teller distribution by beta-decay study. Finally, the calculation reveals the strength of the $M1$ transitions corresponding to the radiative decay of the IAS populated by beta decay and the necessity of gamma detection for a complete experimental survey.

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nucleus	⁴⁴ Cr	⁴⁶ Mn	⁴⁸ Fe	⁵⁰ Co	⁵² Ni	⁵⁶ Zn
$S^+ = \sum_i GT_i^+$	12	12.89	13.26	14.68	15.33	16.69
$S_{renorm.}^+$	6.50	6.98	7.18	7.95	8.30	9.04
$S_{exp.}^+$ (from $S_{exp.}^-(p, n)$)					5.9 ± 1.5^a	9.9 ± 2.4^b

TABLE I. Total value of the calculated β^+ GT transition strength for $T_z = -2$ fp -shell nuclei and renormalized value obtained with the standard quenching factor.

^a Reference [10]

^b Reference [11]

nucleus	⁴⁴ Cr	⁴⁶ Mn	⁴⁸ Fe	⁵⁰ Co	⁵² Ni	⁵⁶ Zn
$t_{1/2}^{fermi}$ (ms)	132	104	98	83	76	56
$t_{1/2}^{GT}$ (ms)	83	39	114	40	146	42
$T_{1/2}^{tot}$ (ms)	51	29	53	27	50	24
$T_{1/2}^{exp}$ (ms)	53_{-3}^{+4a}	41_{-6}^{+7a}	44 ± 7^b	44 ± 4^b	38 ± 5^c	-
$t_{1/2}^{GT}$ (ms)	119	15	60	47	77	83
(Möller) ^d						
$T_{1/2}$ (ms)	83	53	48	36	35	24
(Tashibana) ^e						

TABLE II. The Fermi and GT components of the calculated half-lives are given with the global value. The available experimental values are also presented as well as, in the last two lines, the predicted values from the QRPA model and the gross theory.

^a Reference [20]

^b Reference [23]

^c Reference [19]

^d Reference [27]

^e Reference [26]

nucleus	^{44}V	^{46}Cr	^{48}Mn	^{50}Fe	^{52}Co	^{56}Cu
E_γ (MeV)	2.12	-	2.57	-	2.02	1.81
$B(M1)(\mu_N^2)$	5.57	-	2.87	-	1.87	1.58
Γ (eV)	0.627	-	0.58	-	0.18	0.11
τ_m (fs)	1.06	-	1.15	-	3.63	5.99
Γ/Γ_m (W.u.)	3.12	-	1.40	-	1.04	0.88

TABLE III. Radiative decay calculated in the daughter of the even-even emitter. Calculated transitions strength are also reported (in μ_N^2 and W.u.) with corresponding width and meanlife values.

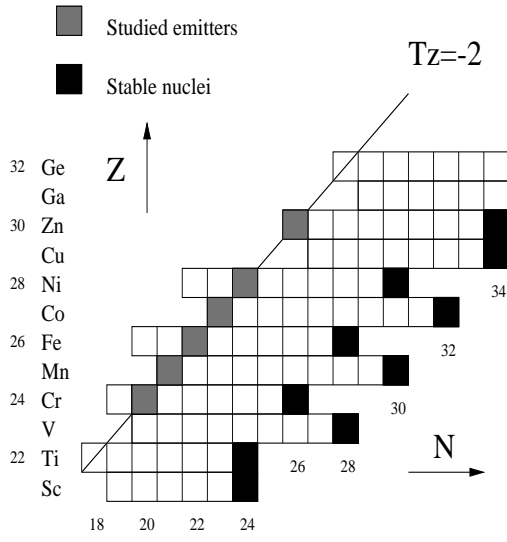


FIG. 1. Proton drip-line and studied nuclei

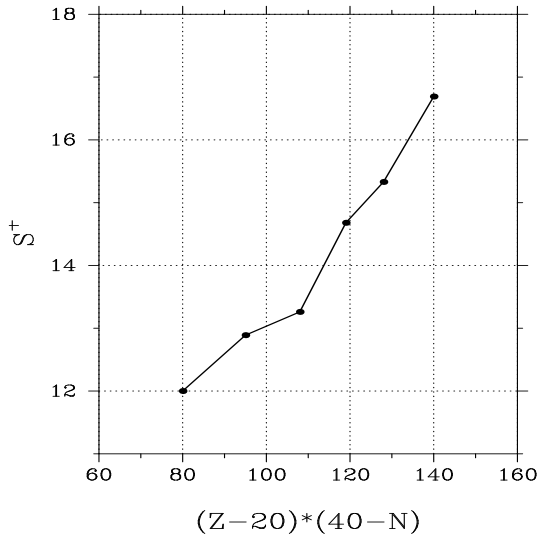


FIG. 2. S^+ as function of number of particles and holes in the valence space.

FIG. 3. Theoretical Gamow-Teller strength in units of GT sum rule (x-axis in MeV).

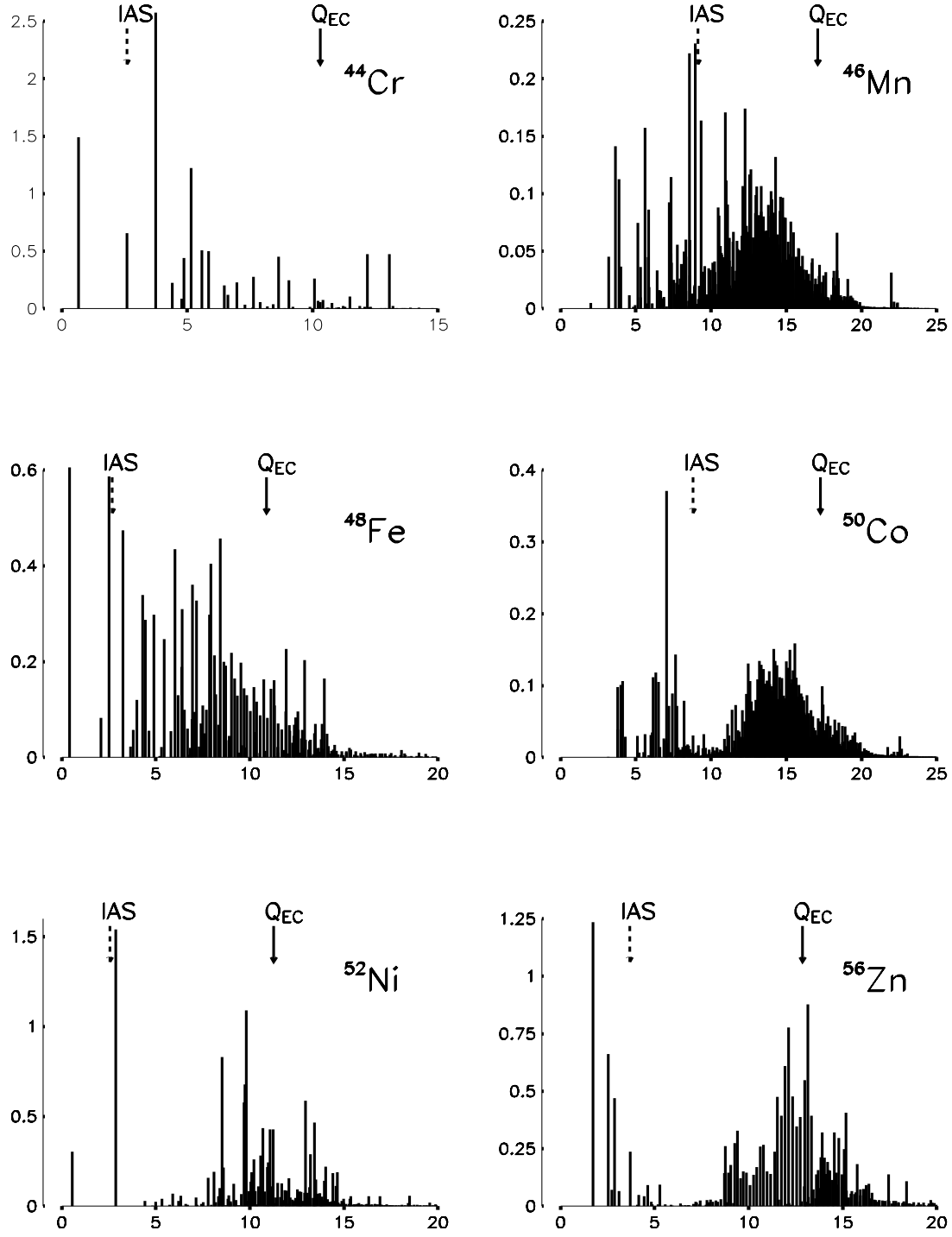


FIG. 4. Theoretical branching ratio (x-axis in MeV, note the change of scale with FIG. 3).

